Chapter 4
Probability

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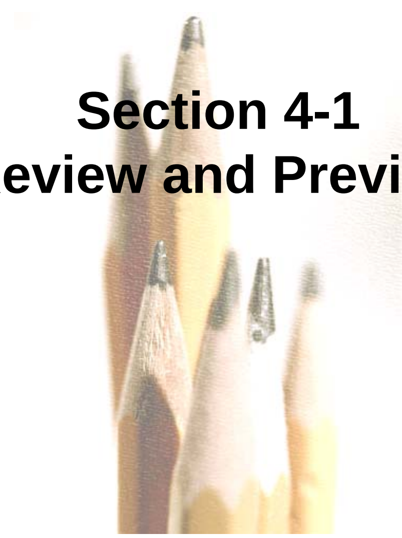
Section 4-1
Review and Preview



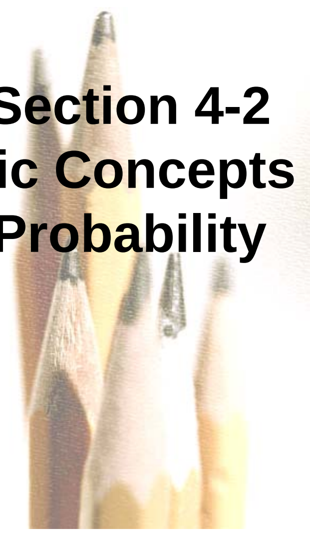
Preview

Rare Event Rule for Inferential Statistics:

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

Statisticians use the rare event rule for inferential statistics.

Section 4-2
Basic Concepts of Probability



Part 1

Basics of Probability

Events and Sample Space

- **Event**

  any collection of results or outcomes of a procedure

- **Simple Event**

  an outcome or an event that cannot be further broken down into simpler components

- **Sample Space**

  for a procedure consists of all possible simple events; that is, the sample space consists of all outcomes that cannot be broken down any further

Notation for Probabilities

P - denotes a probability.

A, *B*, and *C* - denote specific events.

P(*A*) - denotes the probability of event *A* occurring.

Basic Rules for Computing Probability

Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure, and count the number of times event *A* actually occurs. Based on these actual results, *P*(*A*) is approximated as follows:

$$P(A) = \frac{\text{\# of times } A \text{ occurred}}{\text{\# of times procedure was repeated}}$$

Basic Rules for Computing Probability - continued

Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has n different simple events and that each of those simple events has an equal chance of occurring. If event A can occur in s of these n ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

Basic Rules for Computing Probability - continued

Rule 3: Subjective Probabilities

$P(A)$, the probability of event A, is estimated by using knowledge of the relevant circumstances.

Law of
Large Numbers

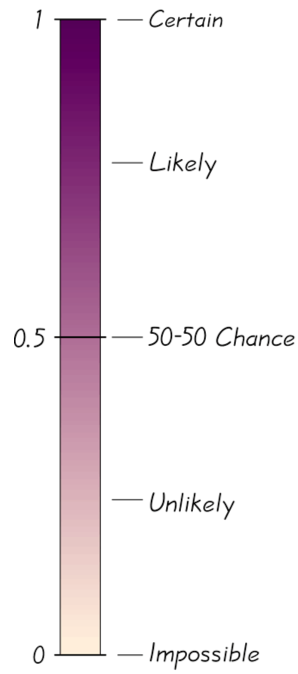
As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Probability Limits

Always express a probability as a fraction or decimal number between 0 and 1.

- **The probability of an impossible event is 0.**
- **The probability of an event that is certain to occur is 1.**
- **For any event *A*, the probability of *A* is between 0 and 1 inclusive.**
  That is, $0 \le P(A) \le 1$.

Possible Values for Probabilities



Complementary Events

The complement of event *A*, denoted by $\bar{A}$, consists of all outcomes in which the event *A* does not occur.

Rounding Off Probabilities

When expressing the value of a probability, either give the exact fraction or decimal or round off final decimal results to three significant digits. (*Suggestion*: When a probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.)

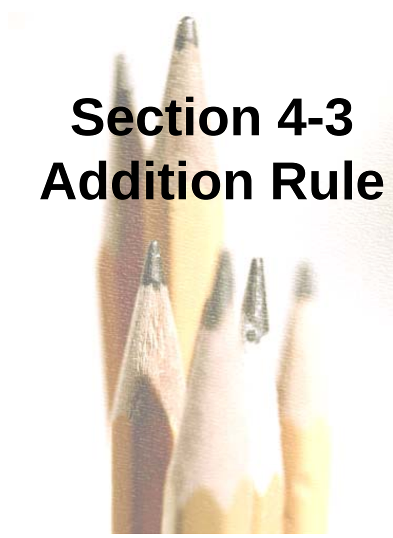
Recap

In this section we have discussed:

- Rare event rule for inferential statistics.
- Probability rules.
- Law of large numbers.
- Complementary events.
- Rounding off probabilities.

Examples

4.2: #5, 9, 12, 13, 15, 24



Section 4-3
Addition Rule

Key Concept

This section presents the addition rule as a device for finding probabilities that can be expressed as *P*(*A* or *B*), the probability that either event *A* occurs or event *B* occurs (or they both occur) as the single outcome of the procedure.

The key word in this section is "or." It is the ***inclusive or***, which means either one or the other or both.

Compound Event

Compound Event

any event combining 2 or more simple events

Notation

P(*A* or *B*) = *P* (in a single trial, event *A* occurs or event *B* occurs or they both occur)

General Rule for a Compound Event

When finding the probability that event *A* occurs or event *B* occurs, find the total number of ways *A* can occur and the number of ways *B* can occur, but find that total in such a way that no outcome is counted more than once.

Compound Event

Formal Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

where *P*(*A* and *B*) denotes the probability that *A* and *B* both occur at the same time as an outcome in a trial of a procedure.

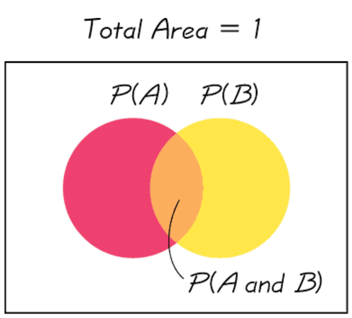
Compound Event

Intuitive Addition Rule

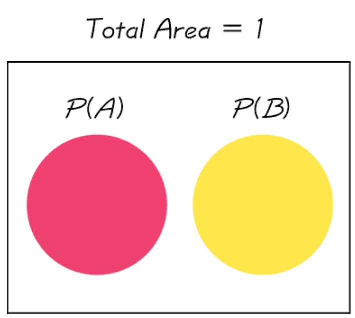
To find *P*(*A* or *B*), find the sum of the number of ways event *A* can occur and the number of ways event *B* can occur, adding in such a way that every outcome is counted only once. *P*(*A* or *B*) is equal to that sum, divided by the total number of outcomes in the sample space.

Disjoint or Mutually Exclusive

Events *A* and *B* are disjoint (or mutually exclusive) if they cannot occur at the same time. (That is, disjoint events do not overlap.)



Venn Diagram for Events That Are Not Disjoint



Venn Diagram for Disjoint Events

Complementary Events

$P(A)$ and $P(\bar{A})$ are disjoint

It is impossible for an event and its complement to occur at the same time.

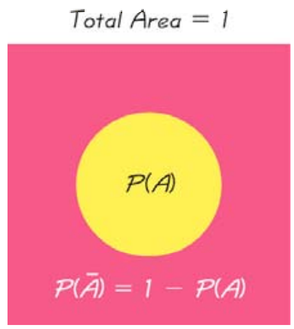
Rule of Complementary Events

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

Venn Diagram for the Complement of Event *A*



Recap

In this section we have discussed:

- Compound events.
- Formal addition rule.
- Intuitive addition rule.
- Disjoint events.
- Complementary events.

Examples

4.3: #6, 11, 13, 16, 18

Section 4-4 Multiplication Rule: Basics

Key Concept

The basic multiplication rule is used for finding $P(A \text{ and } B)$, the probability that event A occurs in a first trial and event B occurs in a second trial.

If the outcome of the first event A somehow affects the probability of the second event B, it is important to adjust the probability of B to reflect the occurrence of event A.

Notation

$P(A \text{ and } B) =$

P(event A occurs in a first trial and event B occurs in a second trial)

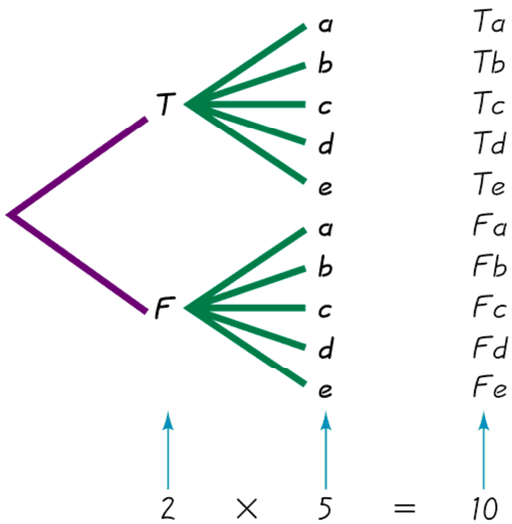
Tree Diagrams

A **tree diagram** is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are sometimes helpful in determining the number of possible outcomes in a sample space, if the number of possibilities is not too large.

Tree Diagrams

This figure summarizes the possible outcomes for a true/false question followed by a multiple choice question.

Note that there are 10 possible combinations.



Conditional Probability
Key Point

We must adjust the probability of the second event to reflect the outcome of the first event.

Conditional Probability
Important Principle

The probability for the second event *B* should take into account the fact that the first event *A* has already occurred.

Notation for Conditional Probability

***P*(*B*|*A*) represents the probability of event *B* occurring after it is assumed that event *A* has already occurred (read *B*|*A* as "*B* given *A*.")**

Dependent and Independent

Two events *A* and *B* are independent if the occurrence of one does not affect the *probability* of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If *A* and *B* are not independent, they are said to be dependent.

Dependent Events

Two events are dependent if the occurrence of one of them affects the *probability* of the occurrence of the other, but this does not necessarily mean that one of the events is a *cause* of the other.

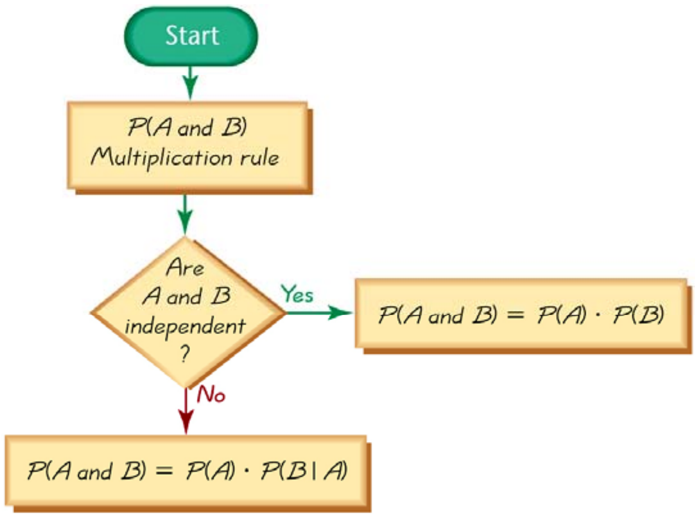
Formal Multiplication Rule

- $P(A \text{ and } B) = P(A) \bullet P(B|A)$
- **Note that if A and B are independent events, P(B|A) is really the same as P(B).**

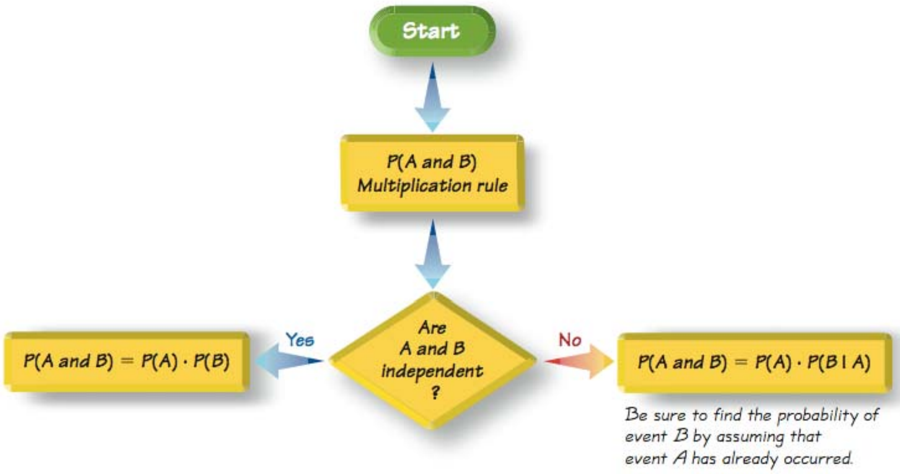
Intuitive Multiplication Rule

When finding the probability that event ***A*** occurs in one trial and event ***B*** occurs in the next trial, multiply the probability of event ***A*** by the probability of event ***B***, but be sure that the probability of event ***B*** takes into account the previous occurrence of event ***A***.

Applying the Multiplication Rule



Applying the Multiplication Rule



Caution

When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.

Multiplication Rule for Several Events

In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

Principle of Redundancy

One design feature contributing to reliability is the use of redundancy, whereby critical components are duplicated so that if one fails, the other will work. For example, single-engine aircraft now have two independent electrical systems so that if one electrical system fails, the other can continue to work so that the engine does not fail.

Summary of Fundamentals

- In the addition rule, the word "or" in $P(A \text{ or } B)$ suggests addition. Add $P(A)$ and $P(B)$, being careful to add in such a way that every outcome is counted only once.
- In the multiplication rule, the word "and" in $P(A \text{ and } B)$ suggests multiplication. Multiply $P(A)$ and $P(B)$, but be sure that the probability of event B takes into account the previous occurrence of event A.

Recap

In this section we have discussed:

- Notation for $P(A \text{ and } B)$.
- Tree diagrams.
- Notation for conditional probability.
- Independent events.
- Formal and intuitive multiplication rules.